2009:

1) $4x+3y=65$
$3(x+5)+4(y-5)=63$

$4x+3y=65$
$3x+15+4y-20=63$

$4x+3y=65$
$3x+4y=68$

2)
a) sum $3:\frac{1}{3}-\frac{3}{4}=\frac{1}{12}=\frac{7}{3\times4}$

sum $4:\frac{5}{4}-\frac{4}{5}=\frac{9}{20}=\frac{9}{4\times5}$

b) sum: $\frac{2n+1}{n\times(n+1)}$

sum $100=\frac{201}{100(101)}=\frac{201}{10100}$

c)
i) $2n+1=1805$
$2n=1804$
$n=902$

ii) $n(n+1)=1806$
$n^2+n-1806=0$
$(n+43)(n-42)=0$
$n=42,\ n\neq-43$
42

3)
a) copper: $.75(6.5)=4.875$ grams $=0.004875$ kg copper

Nickel: $.25(6.5)=1.625$ grams $=0.001625$ Kg nickel

Total Value $=3.70(0.004875)+17.50(0.001625)$
$=0.04638125$

0.05
5 pence

b) 0.0065 kilograms

$17.50(0.0065) = 0.11375$

11 pence

c) $3.70(6.5\frac{x}{100}) + 17.50(6.5\frac{y}{100}) = .10$ (x: copper, y: Nickel)

$3.70(0.0065x) + 17.50(0.0065y) = .10$ (kilograms) ①

$0.02405\frac{x}{100} + 0.11375\frac{y}{100} = .10$

$x + y = 100$ ②

$x + y = 100$

let x be amount of copper

let y be amount of Nickel

$\Rightarrow .0002405x + 0.0011375y = .10$

Solving simultaneously:

$2405x + 11375y = 1000000$

$2405x + 2405y = 240500$

$8970y = 759500$

$y = 84.67\%$

$x = 15.32887402$

$y =$

$x = 100 - y = 15.33\%$

Copper: 15.3%, Nickel: 84.7%

4) $y = \frac{a^3}{x^2+4}$

a) $a = 2, x = 5$

$y = \frac{8}{25+4} = \frac{8}{29}$

b) $x = 1.5, y = 6$

$6 = \frac{a^3}{1.5^2+4}$

$a = (37.5)^{1/3}$

$a = 3.347$

c) $a = 6, y = 12$

$12 = \frac{6^3}{x^2+4}$

$12x^2 + 48 = 216$

$12x^2 = 168$

$x^2 = 14$

$x = 3.742$

a)

5) $\angle CBF = 360 - (90 + 90 + 50)$
$= 130°$

$\angle ABF = 180 - 130° = 50°$

$\angle BAY$ & $\angle BYA = 65°$

$\angle FDE = 130°$

$\angle DFE$ & $\angle DEF = 25°$

$\angle AFE = 360 - (65 + 90 + 25)$
$= 180$

b) $\angle CBD = 180 - x$

$\angle ABF = x$

$\angle BAY$ & $\angle BYA = \frac{180 - x}{2}$

$\angle FDE = 180 - x$

$\angle DFE$ & $\angle DEF = \frac{x}{2}$

$\angle AFE = 360 - \left(\frac{180 - x}{2} + 90 + \frac{x}{2}\right) = 360 - \left(\frac{180}{2} + \frac{180}{2}\right) = 360 - 180 = 180$

$\therefore$ independent of x

6)

a) $V = 9\pi(0.5)^2\sqrt{1 - 0.5^2}$
$= 6.12$

b) When $x = 0$ it means sector comprises of 0% of the circle.

$\therefore$ there is nothing. $\therefore$ Volume $= 0$

When $x = 1$ it means sector comprises of the complete circle. A circle is a 2D plane figure which has no volume. Volume is the amount of space occupied by an object. A cone can only be made out of sectors of a circle, not a full circle. Volume only exists for solids, ie cones, cubes, spheres etc

c) When $x = 0.2$, $y = 9\pi(0.2)^2\sqrt{1-0.2^2} = 1.108$

When $x = 0.3$, $y = 9\pi(0.3)^2\sqrt{1-0.3^2} = 2.427$

" $x = 0.4$, $y = 4.146$

$x = 0.6$, $y = 8.143$

$x = 0.7$, $y = 9.894$

$x = 0.8$, $y = 10.857$

$x = 0.85$, $y = 10.761$

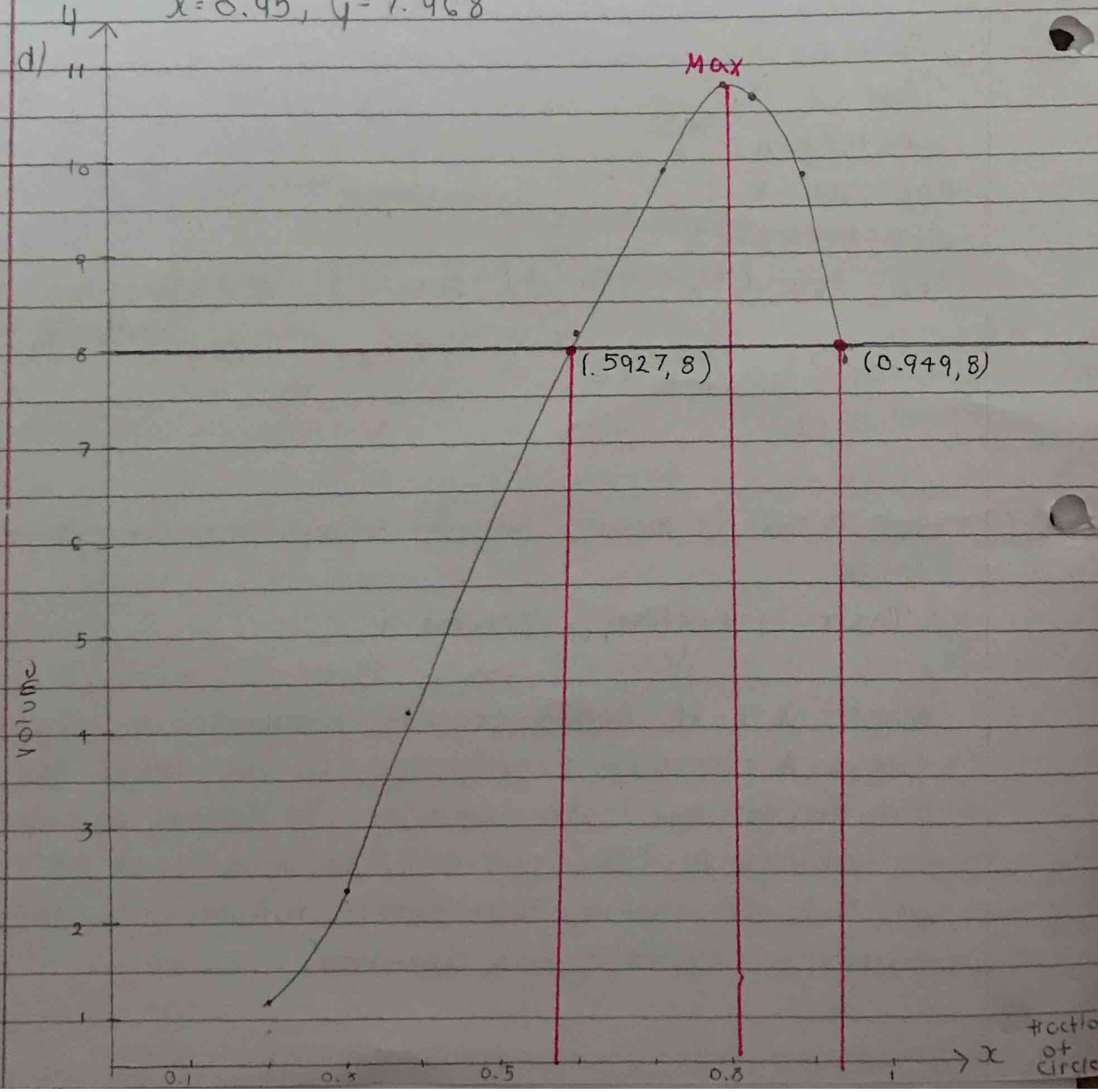
$x = 0.9$, $y = 9.983$

$x = 0.95$, $y = 7.968$

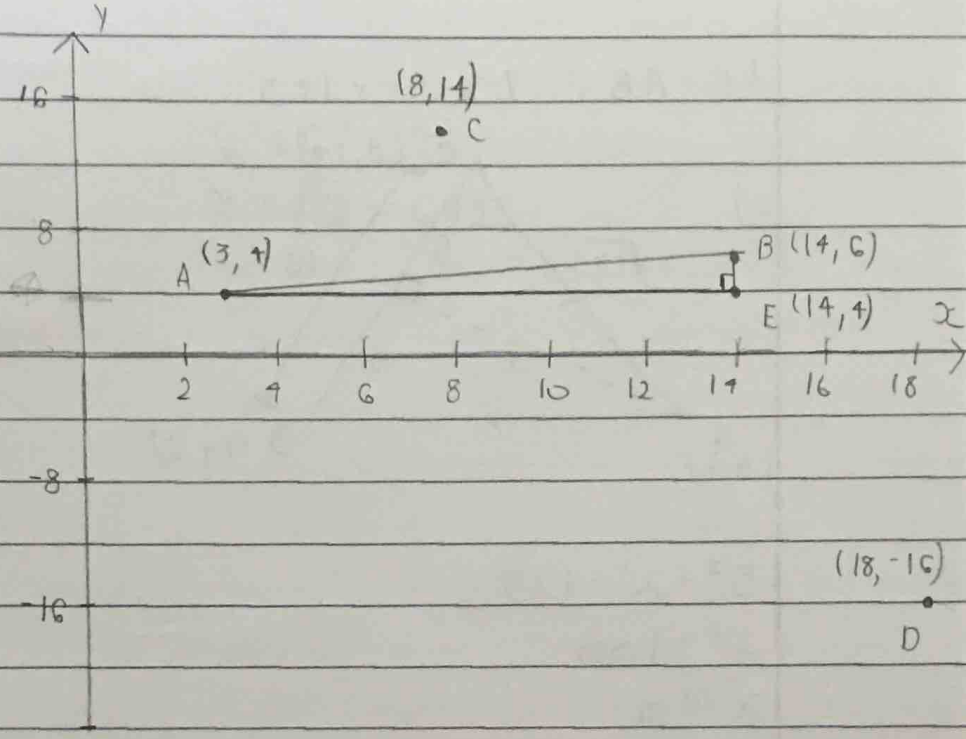
e) $x = 0.59$, $x = 0.95$

f) max when $x = 0.816$

d)

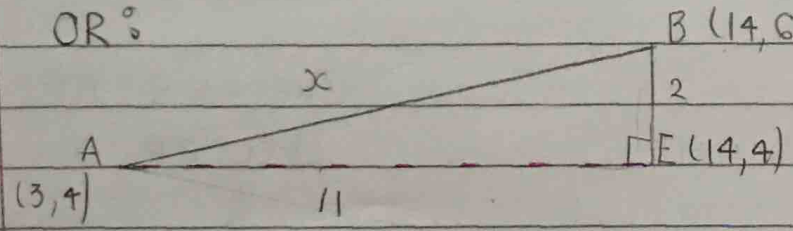


7)



a) $AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{(3-14)^2 + (4-6)^2} = \sqrt{121+4} = \sqrt{125}$

OR:



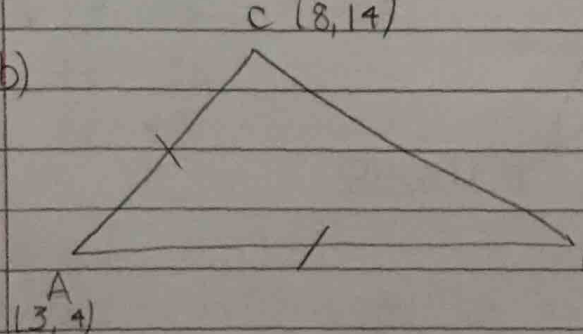
$AE^2 + EB^2 = AB^2$

$11^2 + 2^2 = AB^2$

$AB^2 = 125$

$AB = \sqrt{125}$

b)

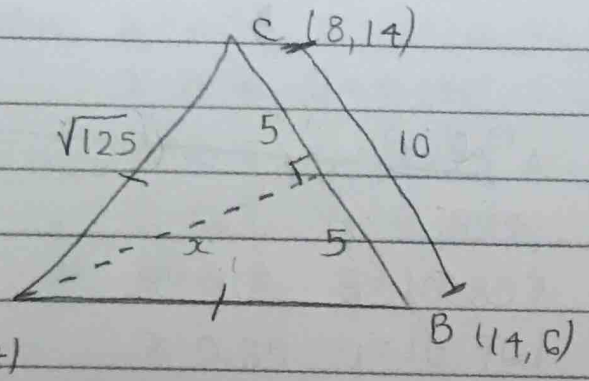


Distance $= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$AC = \sqrt{(8-3)^2 + (14-4)^2} = \sqrt{25+100} = \sqrt{125}$, $AB = \sqrt{(14-3)^2 + (6-4)^2} = \sqrt{121+4} = \sqrt{125}$, $BC = \sqrt{(8-14)^2 + (14-6)^2} = \sqrt{36+64} = 10$

AC = AB ∴ Isosceles

c)



$5^2 + x^2 = 125$

$x^2 = 100$

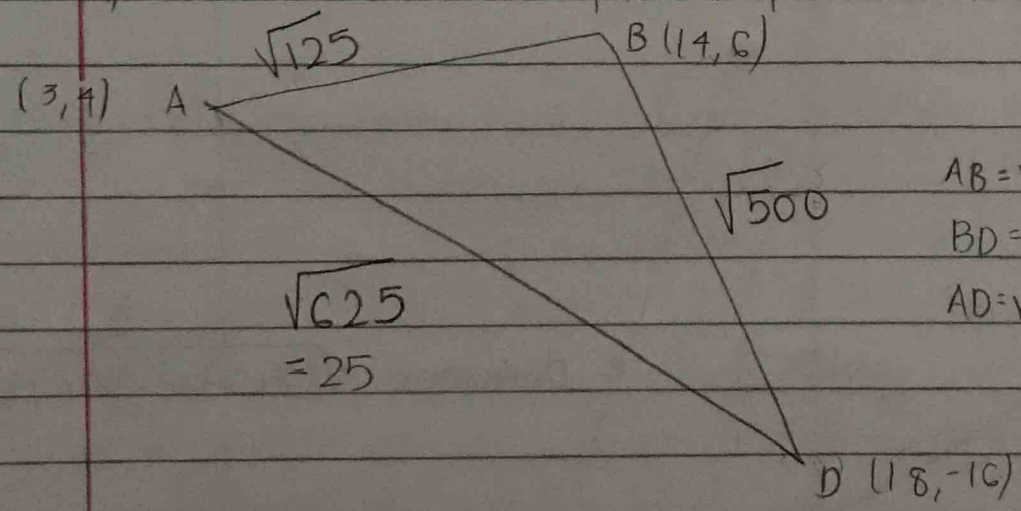
$x = 10$

Area $= \frac{1}{2}$ base x height

$= \frac{1}{2}(10)(10)$

$= 50$

d) Show ABD is right angled



$AB = \sqrt{125}$

$BD = \sqrt{(-4)^2 + 22^2} = \sqrt{500}$

$AD = \sqrt{(-15)^2 + (20)^2} = \sqrt{625}$

$AB^2 + BD^2 \stackrel{?}{=} AD^2$

$125 + 500 = 625$

∴ right angle triangle by pythagoras's theorem

8)

a) $I : \frac{3}{4}(36)\pi = 27\pi$

$II : 6\sqrt{2}\,x$

$6^2 + 6^2 = x^2$

$x = \sqrt{72} = 6\sqrt{2}$

$27\pi = 6\sqrt{2}\,x$

$x = \frac{27\pi}{6\sqrt{2}}$

$x = 9.996$

b) $I : \frac{3}{4}(12\pi) = 9\pi$

$II : 2x + 6\sqrt{2} + 6\sqrt{2} = 2x + 12\sqrt{2}$

$9\pi = 2x + 12\sqrt{2}$

$x = \frac{9\pi - 12\sqrt{2}}{2}$

$x = 5.652$

9)

a) $T_5 = 15$

$T_6 = 21$

$T_7 = 28$

$T_8 = 36$

$T_9 = 45$

Formula $= \frac{n(n+1)}{2}$

b)

i) $T_1^2 + T_2^2 = 1 + 9 = 10 = T_4$ 4 = 1st # Squared

ii) $T_2^2 + T_3^2 = 9 + 36 = 45 = T_9$ 9 = 1st # Squared

c) $T_3^2 + T_4^2 = T_{16} = \frac{16(17)}{2} = 136$

d)

i) $T_{10}^2 + T_{11}^2 = T_{121} \Rightarrow x = 121$

ii) $T_4^2 + T_{4+1}^2 = T_{81}$

$(4+1)^2 = 81$

$(4+1)^2 = 9^2$

$4+1 = 9$

$4 = 8$

$T_8^2 + T_9^2$

$x = 3.742$